## Tutorial 1 (Functions)

**(You should practise writing proper steps.)**

1.(a) $f(x) = x^2, \ x > 2, \quad g(x) = x^2, \ x \in [0,4]$

      Explain why $f$ and $g$ are different functions.

  (b) $f(x) = x + \sqrt{x-1}, \quad g(u) = u + \sqrt{u-1}$ . Is it true that $f = g$ ?

  (c) $f(x) = \dfrac{x^2 + x}{x+1}, \quad g(x) = x$ .  Is it true that $f = g$ ?

2. $f(x) = x - 1, \quad g(x) = \dfrac{1}{x+1}$. Find

(i) $\quad (f \circ g)(2), \ (g \circ f)(2), \ (f \circ f)(2), \ (g \circ g)(2)$

(ii) $\quad (f \circ g)(\tfrac{1}{2}), \ (g \circ f)(\tfrac{1}{2}), \ (f \circ f)(\tfrac{1}{2}), \ (g \circ g)(\tfrac{1}{2})$

(iii) $\quad (f \circ g)(x), \ (g \circ f)(x), \ (f \circ f)(x), \ (g \circ g)(x)$

    [For (iii), write the expression in $x$, and simplify.]

(iv) Find the domains of $f, g, f \circ g, g \circ f, f \circ f, g \circ g$ .

3. Let $f(x) = x - 2, \ g(x) = \sqrt{x}, \ h(x) = x^3$ and $j(x) = 3x$. Express the following as a composite (i.e., composition function) involving one or more of $f$, $g$, $h$ and $j$.

    (a) $\quad y = 3x - 2$         (b) $y = \sqrt{x^3 - 2}$     (c) $\quad y = 3\sqrt{x - 2}$

4. Find the domain of each function. Write the domain in the form of an interval or union of intervals.

| | |
|---|---|
| $f(x) = \dfrac{4}{2-x}$ | |
| $g(x) = \sqrt{9 - x^2}$ | |
| $h(x) = \sqrt{5x + 1}$ | |
| $m(u) = \dfrac{4 - u^2}{2 - u}$ | |
| $n(t) = \dfrac{1}{t^2 + 2t - 3}$ | |
| $p(w) = \dfrac{w + 1}{4 - \sqrt{w^2 - 9}}$ | |

5. Complete the following table.

| $f(x)$ | $g(x)$ | $(f \circ g)(x)$ |
|---|---|---|
| $2x+4$ | $3x$ | ? |
| $\sqrt{x+3}$ | $x^2+x$ | ? |
| $\dfrac{x}{x+1}$ | $\dfrac{x}{x+1}$ | ? |
| $1+\dfrac{1}{x}$ | ? | $x$ |
| ? | $\dfrac{1}{x}$ | $x$ |
| $\sqrt{x}$ | ? | $|x|$ |

6. Sketch the graphs of the following functions.

   (a)   $y=\sqrt{2x+1}$        (b)  $y=-\sqrt{2x+1}$        (c)  $y=(x-2)^2+1$

   (d)   $y=(x+3)^2-2$      (e)   $y=3|x|$        (f)  $y=|2x+1|$

   (g)  $g(x)=\begin{cases} 3-x & \text{if } x \le 3 \\ 2x+1 & \text{if } x > 3 \end{cases}$      (h) $h(x)=\begin{cases} -x & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

   (i)   $p(x)=2+\sqrt{x-1}$          (j)   $q(x)=2-\sqrt{x}$

   (k)  $r(x)=\dfrac{1}{x}$                (l)  $s(x)=\dfrac{1}{x-2}+2$

7. Determine if the function is even, odd, or neither. Justify your answer. [This means "show steps to arrive at the conclusion".]

  [Note that the sine function is odd and the cosine function is even.]

(a) $f(x)=x^2+\cos x$     (b) $g(x)=x-\sin x$     (c) $h(x)=x^5+x^3-2x$

(d) $k(x)=\dfrac{x}{x+1}$       (e) $m(x)=\dfrac{x^2+1}{x^3+x}$       (f) $n(x)=\dfrac{x^2}{x^2+1}$

8. Given the graph of $f(x)$ shown, sketch an approximate graph each of the following. If you wish to graph more than one on the same axes, label each with the letter of the problem or with its equation.
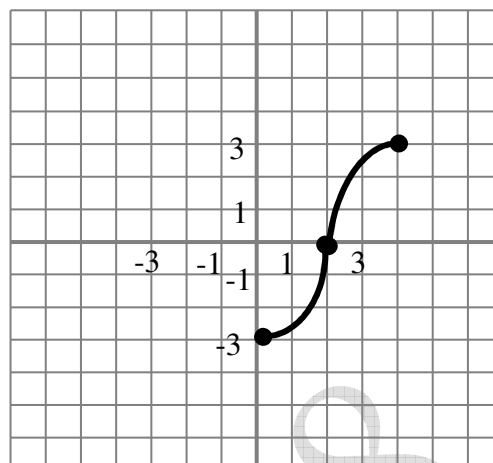
   A. f(x – 3)       B. f (x) - 2

   C. – f (x)       D. f(x + 1)

   E. f(– x)       F. | f(x) |



9. For $f(x) = 2x^2 – 8x – 10$,

 (a) does the parabola open up or down?

 (b) find:

   (i) the coordinates of the vertex     (ii)  the $x$-intercept(s)

   (iii) the y-intercept          (iv) equation of axis of symmetry

10. Graph each of the following.  If you wish to graph more than one on the same axes, label each with the letter of the problem or with its equation.

   A. $f(x) = \sqrt{x}$        B. $g(x) = \sqrt{-x}$        C. $h(x) = \sqrt{x} - 2$

   D. $j(x) = -\sqrt{x}$        E. $k(x) = \sqrt{x-2}$        F. $m(x) = -2\sqrt{x+3} + 1$

(nby, Jun 2017)